Chapter 15.5

Glossary:

For a function

are the **independent variables**

are the **intermediate variables**

is the **dependent variable**

It is helpful to use a **tree diagram** to remember the Chain Rule

The **Implicit Function Theorem** (ℝ2) states that if a function is defined on a disk containing , where , and and are continuous on the disk, then defines as a function of near , the derivative of which found by **Implicit Differentiation**.

The **Implicit Function Theorem** (ℝ3) states that if a function is defined within a sphere containing , where , and , and are continuous inside the sphere, then defines as a function of near , the partial derivatives of which found by **Implicit Differentiation**.

**The Chain Rule (Case 1):** Suppose that is a differentiable function of , where are both differentiable functions of . So is a function of and

**The Chain Rule (Case 2):** Suppose that is a differentiable function of , where are both differentiable functions of . So is a function of and

**The Chain Rule (General):** Suppose a function is a differentiable function of , where each . So is a function of , and

for each

**Implicit Differentiation:** Suppose, for a differentiable function , we define a function , that is, . If is differentiable, we can apply Case 1 of the Chain Rule:

Solving for , we get

**Now**, suppose for a differentiable function , we define a function , that is, . If is differentiable, we can apply the Chain Rule:

We can do the same for . Solving for , and for we get

and

15.7 Maximum and Minimum Values

Important Ideas:

**Definition**: A function of two variables has **local maximum at (a,b) if**  when f(x,y) is near (a,b). This means that for all points (x,y) in some disk with center (a,b). The number f(a,b) is called a **local maximum value.** If when (x,y) is near (a,b), then f has **a local minimum** at (a,b) and f(a,b) is a local minimum value.

**Theorem**:

* If f has a local maximum or minimum at (a,b) and the first-order partial derivatives of f exist there, then **.**

**Second Derivative Tests**: Suppose the second partial derivatives of f are continuous on a disk with center (a,b), and suppose that [that is, (a,b) is a critical point of f]. Also, let

1. If D>0 and , then f(a,b) is a local minimum.
2. If D<0 and , then f(a,b) is a local maximum.
3. If D<0, then f(a,b) is not a local maximum or minimum.

In case 3, the point f(a,b) is called a **saddle point**, where the graph of f crosses its tangent plane at (a,b).

Note: If D=0, no information is given from the test results; f could be a local max or min or a saddle point.

**Critical Point**: Occurs when if , or if one of these partial derivatives does not exist.

**Definition:** If f is continuous on a closed, bounded set D in , then f attains an absolute maximum value and an absolute minimum value at some points in D.

To find the absolute maximum and minimum values of a continuous function f on a closed, bounded set D:

1. Find the values of f at the critical points of f in D.
2. Find the extreme values of f on the boundary of D.
3. The largest of the values from steps 1 and 2 above is the absolute maximum value; the smallest of these values is the absolute minimum value.